Information technology for modeling of optimization process of financial providing innovative activities of small business

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ABSTRACT

The article outlines a number of the most important factors, which at the same time are sources of financing for innovations of small enterprises, and based on them, a mathematical model of optimization of financial support of innovative activity of small business has been developed. For simplicity of practical application, the developed model does not take into account multiplicative and synergistic effects, as well as ignores the time factor, so the model is considered static. Also, the linear dependence version for this multivariate model is reviewed. Taking into account the factors included in the functional dependence of small business are varied in their weight, we introduced a weighting factor k, to determine the magnitude of which used the method of active examination, and to verify the accuracy of expert data, the calculation of the coefficient of concordance and statistical error. Taking into consideration the expert-defined values of weighting factor, the initial actual for Ukraine financial model for the innovative activity of small business was originally obtained. The resulting model has been refined to take into account China's experience by adjusting the weight coefficients to the value predicted by the experts. This will allow getting a significant economic effect both at micro and macro levels. Also, for the "bank loans" factor, a theoretical-game model of the interaction of financial structures and subjects of innovation activity has been developed, which proves that, for the financial structures defined in the model, it is advantageous to finance the innovative start-up of small businesses.

Keywords: Times Roman, image area, acronyms, references

1. INTRODUCTION

The mechanism of financial support for economic activity has a complex structure, which includes organizational (organization, development and implementation of innovations), economic (regulation, management, planning and implementation of innovations) and financial components, which are interdependent and together allow to achieve the desired effect. Such mutual dependence presupposes the close interaction of business entities, state authorities and local governments, at the disposal of which there are financial, material, mineral substance materials, intellectual and informative resources for maintenance of effective economic activity, which in the present conditions is impossible without the introduction of innovations.

It should be noted that in recent years, the issue of financing start-up projects is becoming increasingly important, especially in small businesses, both globally and in Ukraine in particular. Private entrepreneurs and entrepreneurs-innovators have not so many real ways to fund their projects. Of course, there are state programs for supporting innovation small business, private grants, but participation in them requires time consuming and, as a rule, takes place on a competitive basis and at certain times for a specific innovation direction (environmental, social, technological, etc.). In addition, such state or local programs are often sufficiently limited in funding [1]. It is clear that the fastest way to get the necessary funding is to apply for bank loans, but the problem is high interest rates even for traditional projects, not to mention the rates on credit resources provided by banks for the development and implementation of risky innovation projects, as well as because, as a rule, start-up developers are not in a position to present commercial banks with a pledge they require before giving a loan by checking the financial capacity of the borrower.

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In general, it should be noted that both state and non-state financial institutions in Ukraine are very weakly oriented towards investing in innovations, which is why domestic entrepreneurs are forced to look for money while addressing issues of financing innovation development, referring to other non-bank financial institutions.

2. LITERATURE REVIEW

Problems of financial support for innovation activity are researched in the scientific works of such Ukrainian scholars as S. S. Varnaliy [2,6], D.S. Ocheretnyi [3], L. M. Savchuk [5], L. M. Vasilieva [7], Ya.B. Dropa [8], Kh. P. Danylkiv [8], V.V. Kosteckyi [9], A.M. Butov [9], M. Pyvovarov [10], T. Shcherbata etc., which reveals the issues of functioning of small enterprises, in particular, the principles of management and stimulation of the development of innovation activity of small enterprises in Ukraine and the world are considered, the methods of their financial support and the factors influencing him, analyzes trends and prospects of financial support for innovation activities of small enterprises in Ukraine, revealed problems of interaction between financial institutions and small businesses.

The work of foreign scientists such as B. Feld and J. Mendelson [11], J. Loewen [12], V. Kerr [13], etc., which reveals the questions of creation and functioning, is devoted to the presentation of the world experience in the organization of small innovative business, small innovative enterprises, highlights the main ways of financial support for small businesses, discusses the origins of creating the most successful start-ups in the world and methods of their financial support, examines the main sources of financial support for innovative activities of small businesses. At the same time, the development of a quantitative model for optimizing the interaction of commercial financial institutions and small businesses in terms of financing start-up projects is not paid due attention of economists, which is why this important issue remains undiscovered. Although some steps have been taken in the work of the authors [3, 10, 12, 14, 15], the generalized system, on the basis of which a multi-factor model for the financial support of the functioning of a small innovative enterprise could be developed, was not developed.

3. METHODS

In order to develop an effective mathematical model for optimizing the financial support of innovative business activities of small businesses, it is first of all necessary to analyze the existing sources of financial support for business, which are the most widely spread and accessible both in the world and in Ukraine, in particular.

In order to improve the situation in the context of financing innovative activity of small enterprises in Ukraine, venture capital, traditionally oriented towards small and medium business, can play a major role. Other diversified innovation funds (*SIF*), both domestic and foreign, non-state grants, business incubators. Scientific and technical incubators, techno-parks firms, technopolises, etc., which, on preferential terms, ensure the formation of innovative small and medium-sized enterprises. Thus, incubators provide innovative firms with a certain period of premises and necessary equipment, advise them on preferential terms on economic and legal issues, organize advertising, carry out scientific and technical, environmental and commercial expertise of innovative projects, search for investors, make use of their research production.

Venture funds are primarily based on the financing of small, knowledge-intensive businesses. In developed countries, venture companies enjoy significant support from the state and large companies, which is not always profitable to develop new technologies due to the existence of significant risk of loss. Therefore, large companies often refer to venture financing, using it as a tool to expand its own scientific and technical potential. Unfortunately, in Ukraine, venture financing has not yet been properly disseminated and developed, as the government does not pay enough attention to venture capital business, does not encourage venture capital investors to invest in innovative projects, and therefore the activity of venture funds in Ukraine is oriented mainly to trade operations [4, 5].

With regard to state-owned innovation business support programs (*GPSIC*), the effective way of helping small innovative enterprises in Ukraine could be, in our opinion, the provision, at relatively low interest rates and long terms of return, of funds to private venture firms, which in turn would be directly financing the prospective small businesses. In certain cases, these funds may be provided on an irrevocable basis. This approach has been applied in Finland, where the «Suomen Teollisuus-Sijoitus OY» state fund was established in 1994 and in Germany, were in 1995 a scheme called «Beteiligungskapital fur Kleine Technologieunternehmen» was developed, which financed small firms, producing innovative products or services in partnership with private sector companies. In Denmark, the government also provides loans to small firms for the development of technology-oriented projects. It is stipulated that in the event of failure, the debts will be cancelled. Approximately the same scheme for providing loans for technical development of enterprises exists in the Netherlands. It involves the issuance of ten-year loans to small companies, which in case of a technical or

commercial failure may not be returned to the state. And the largest shareholder is the Swedish government, which provides loans to start-up companies for a period of 6-10 years. These loans are interest-free for the first two years and do not involve repayment of principal during the first four years [3, 5]. Nor should be forgotten about state support for small businesses in the form of such actions as obtaining grants for individual innovative projects at the micro level and the introduction of preferential fiscal policy for innovative enterprises at the macro level – levels of state as a whole.

The limited means of state and regional support for small business increases the role of such alternative financial and credit development instruments of domestic small business entities such as bank loans (*CB*), franchising (*F*), fundraising (*FR*) and crowdfunding (*C*).

Bank loans (*CB*) is a necessary element of the functioning of a market economy, and maybe the only quick way to get the necessary funding. But, unfortunately, as was already mentioned, the problem is the establishment of high credit rates by banks, even for traditional commercial projects, and a requirement by the creditor of a borrower's financial capacity in the form of high collateral for the funds provided.

Fundraising (*FR*), as a financial source, is not widely spread in Ukraine today. In most cases, it is associated with philanthropy and social projects. However, it is important to understand that besides this method of attracting resources for commercial projects, search for a financial patron or a donor is also fundraising [16].

As for such a source of financial support as Crowdfunding (*C*), there are crowdfunding platforms for project development, where anyone can invest any amount on their implementation. The most popular crowdfunding platforms in Europe are the following [5, 16]: Crowdfunding International (Netherlands), Ulule (France), Wemakeit (Switzerland), Biggggidia (Ukraine) and Na-Starte.com (Ukraine).

One of the ways of small innovative business effective support is Business Angels partnership (*BA*). Today about 500,000 business angels work in the world [4, 17]. Business Angel is a private investor who invests his own funds in risky, promising projects at the stage of creating a company in exchange for a share in its capital. Angels tend to invest their own funds, unlike venture investors who manage a pool of other people's money in a professional fund [4, 16]. The business angel's funds fill the gap between financing start-ups between the funds of «friends and family» [12], which provide basic funding, and formal venture capital, which is often not interested in small investments. Among the start-ups raised by the angels are billiard airlines, biotechnology, financial, computer, energy companies, etc. Many of them are well known: Xerox, Amazon.com, Ryanair, Apple, Xynergy, Compaq, America Online, Google, E-bay, Innovative Robotics and many others [5,20].

With regard to possible internal sources of financing of small business remain:

- own initial capital (*IC*) – is the financial basis for the creation and development of an enterprise of any form of ownership and organizational-legal form.
- the implementation of licenses for its own innovative product is an innovative product (*L*), including an exclusive, single, non-exclusive and, if necessary, an open license.
- reinvestment of the share of profit (*RP*) received from the sale of the product. It should be noted that for newly created enterprises, this factor at the beginning of their life cycle will be equal to zero, and even further if profitable business of the company it will be positive [21, 22].

After examining the existing sources of financial support for innovative business activities of small businesses, we will highlight the most relevant among them in the form of a multifactorial model for optimizing the financial support for innovatively active enterprises (FS_{IAE}), through the example of small enterprises in the classical hierarchical structure of the objectives tree (Figure 1).

Since the simulation of the financial support of innovation activities of enterprises is often carried out using deterministic, statistical, expert and combined methods, then the basis of the mathematical expression is reflected in Fig. 1 models we use the classical set of factors and their functional dependence, which in general form is represented in the formula (1) [18,23]:

$$X = \{x_1, x_2, ..., x_n\} \rightarrow f \in Ff = \{f_1, f_2, ..., f_n\} \quad (1)$$

where *X* is the set of factors influencing the financial support process, *Ff* is the set of financial support for innovative activities.

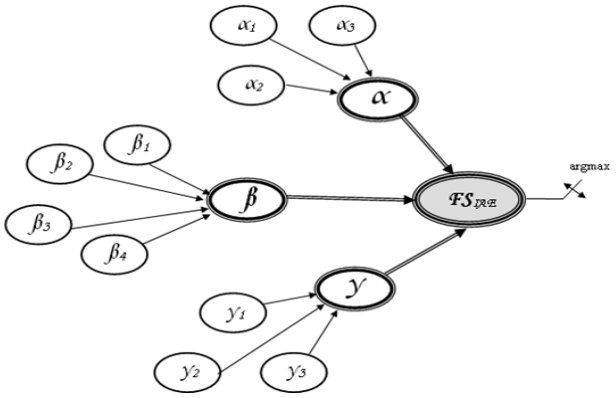

Figure 1. A generalized model for optimizing financial support for innovative activities of small enterprises

For the determination of a specific quantitative weight *k*, the method of active examination was used [19]. The procedure for determining the weight coefficient *k* is the formation of a final evaluation with a strictly increasing continuous function satisfying the condition of unanimity [19, 20]. Experts are expected to report their true thoughts, and if each of the experts is a little mistaken (unknowingly and depending on their qualifications), then the average score will be determined by the formula [24]:

$$k = \mathrm{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \,, \tag{2}$$

Formula (2) is fairly objective and accurately estimates the object, since in the equivalent direct mechanism of communication of reliable information is the balance of Nash for experts, and the final evaluation is the same as in the original mechanism [19, 20].

To simplify the mechanism for evaluating experts and aggregate their results using formula (2), we use the following, developed by the authors, by the method of selection of experts. Let *Ei* be a tuple of the characteristics of *i*-expert, then formula (3) will look like this:

$$\mathrm{E_i} = \langle V_i, B_i, P_i \rangle \,, \tag{3}$$

where V_i – experience of the expert (number of years that worked on the required position), *Bi* – probability of the correct conclusion, $B \in [0,1]$, *Pi* – field of expert knowledge.

Consequently, the requirements for selection of experts will be as follows:

$$\bigcup_{i=1}^{n} P_i = P_{tot.} \,, \tag{4}$$

$$\forall i\colon \mathrm{P_i} \le P_{\mathrm{tot.}} \,, \tag{5}$$

where P_{tot} – a general field for examination (the whole set of expert questions).

$$\forall i\colon B_{\mathrm{i}} \le B_c \,, \tag{6}$$

where B_c – the critical value of probability.

$$\forall i\colon V_{\mathrm{i}} \le V_c \,, \tag{7}$$

where V_c – the critical value of experience.

When working out the calculation of consistency, if necessary, in case of inconsistency of expert opinions, the results of those experts for whom the following requirement is fulfilled are rejected:

$$V_j = \min_i \{V_i\}_{i=1}^{n} \,, \tag{8}$$

If two experts have the same value of *Vi*, then there is an expert with a higher value of B_i. If, however, these values are the same, there remains an expert with a greater value of P_i. To verify the correctness of the expert data, the calculation of the coefficient of concordance was carried out using formulas (9) and (10) [21].

$$\overline{\sum p} = \frac{n \times (m+1)}{2} \tag{9}$$

where n – number of experts, m – number of possible evaluation options.

$$w = \frac{12 \sum \Delta^2}{n^2 (m^3 - m)}, \tag{10}$$

where Δ^2 – deviation from the average amount of ranks squared.

Since expert evaluations were conducted for mutually influential factors, there is a need to check the accuracy of expert calculations for the maximum allowable total error according to formulas (11) – (13) [21].

$$\sigma = \sqrt{\sigma^2}, \tag{11}$$

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \overline{X})^2}{N}, \tag{12}$$

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{N}, \tag{13}$$

where $\overline{X}$ – average value of the indicator, X_i – indicator value, N – the number of indicators.

4. 4. RESULTS AND DISCUSSION

Mathematically presented in Fig. 1 model can be expressed by the following dependencies:

$$\mathrm{FS_{IAE}} = \mathrm{f}(\alpha, \beta, \gamma) \rightarrow \mathrm{argmax} \tag{14}$$

where α – own capital, β – objective capital attracted, which is provided on terms of return through a specified period, with interest (or interest-free) for its use, γ – subjectively attracted capital provided to support the project without its return in the future.

In turn, equity capital is expressed as a functional dependence (15) (first group of factors):

$$\alpha = \mathrm{f}(\alpha_1, \alpha_2, \alpha_3) \tag{15}$$

where α_1 – own initial capital, α_2 – licensing to interested parties, α_3 – reinvesting profits.

Objective attracted capital, which is provided on terms of return through a specified period, with interest (or interest-free) for its use, is a function depending on four main factors of the second group:

$$\beta = \mathrm{f}(\beta_1, \beta_2, \beta_3, \beta_4) \tag{16}$$

where β_1 – government programs to support innovative entrepreneurship, β_2 – partnership with a variety of innovative funds, β_3 – partnership with business angels, β_4 – bank loans.

In this model, we refer to the objectively captive capital only those financial resources that need to be returned with interest, without them (only the body of a loan) or in the form of a percentage of profit, and do not include crowdfunding and fundraising, because the first operates on a royalty-free basis, and fundraising is proposed to be used only to attract a financial patron or donor who also operate on a royalty-free basis.

And, finally, the subjectively attracted capital provided to support the project, without its return in the future, is expressed by the following dependence (third group of factors):

$$\gamma = \mathrm{f}(\gamma_1, \gamma_2, \gamma_3) \tag{17}$$

where γ_1 – fundraising, γ_2 – crowdfunding, γ_3 – franchising.

It should be noted that franchising (γ_3) in the developed model is appropriate to express in the form of two of its main variants. When using the first option (γ_3^1), the transfer of technology and the name of the company to the third party concerned is subject to one-time remuneration. In the second variant (γ_3^2), there is the technology transfer and the name of the company to the third party on the terms of receipt of remuneration in the form of a specified percentage of future revenue or profit from the sale of the transferred innovative product or technology:

$$\gamma_3 = \sum_{i=1}^{n} \mathrm{k}_i \gamma_3^i , \tag{18}$$

$$k_i = 0,1, \tag{19}$$

$$\gamma_3 \in \gamma_3^1, \gamma_3^2 , \tag{20}$$

where γ_3 – total financial resources received from franchising; n – number of firms that have purchased franchising; k – choice of franchising option; γ_3^i – financial resources received from each franchise option.

The integration of formulas (18), (19) and (20) into a single local model of obtaining financial resources for all possible franchising options is expressed in equation (21):

$$\gamma_3 = \sum_{i=1}^{n} \gamma_3^{1,i} + \sum_{j=1}^{m} \gamma_3^{2,j} , \tag{21}$$

where $\sum_{i=1}^{n} \gamma_3^{1,i}$ – financial resources derived from the first version of franchising from all n-numbers i-firms,

$\sum_{j=1}^{m} \gamma_3^{2,j}$ – financial resources derived from the second version of franchising from the entire m-number of j-firms.

Among the factors outlined in the formula (16), it is necessary to highlight such an important factor for Ukraine and other transition economies as β4-bank loans, since bank loans is the fastest way to obtain the necessary funding. The problem lies in high lending rates on ordinary loans and the financial capacity of the lender in the form of a high collateral. For a small business, which, in the vast majority, does not have the necessary financial or material support for a loan, such conditions are an obstacle to the implementation of their innovative ideas [22]. And modern small innovative enterprises are oriented, as a rule, on IT technologies or FinTech projects [5]. In particular, to create start-ups in the IT or FinTech areas, the share of attracted capital in the aggregate of their sources can range from 50 to 100%, depending on the particular project.

Commercial banks, as a rule, are rather conservative regarding the terms of funding for innovative ideas and start-up projects, since there is a high risk of their non-return [4, 5]. But, on the other hand, if an innovative start-up project is successful, then the profits from it exceed the financial costs of tens, hundreds, and sometimes thousands of times [16]. This is the reason why financing a successful start-up project is highly desirable for financial institutions. It should be noted that the development of the theory-game model of the interaction of financial structures and subjects of innovation requires such an optimization of their partnership, which can be proved by methods and tools of game theory.

First of all, start-up in a small business requires a clearly defined condition in which such financing is possible and beneficial to commercial banks. Consequently, the general condition for a commercial bank, in which it will be profitable to finance start-up, can be expressed by the following equation [22]:

$$N_+(\overline{V_c} + \overline{\Delta V}) + N_- \overline{V_c} > \overline{V_c}(N_+ + N_-) + \overline{V_c} q \cdot (N_+ + N_-) \cdot \tau , \tag{22}$$

where N_+ – number of effective projects, units, N_- – number of unsuccessful projects, units, $\overline{V_c}$ – the body of credit, money unit, $\overline{\Delta V}$ – average profit from start-up for the entire period of existence of start-up, money unit, q – annual loan rate, %, τ – the term of the loan, which coincides with the period of existence of start-up, years.

For the economic interpretation of inequality (22) we will carry out a series of mathematical transformations on it:

$$N_+\overline{V_c} + N_+\overline{\Delta V} + N_-\overline{V_c} > N_+\overline{V_c} + N_-\overline{V_c} + \overline{V_c}q\cdot(N_+ + N_-)\cdot\tau, \tag{23}$$

$$N_+\overline{\Delta V} > +\overline{V_c}q\cdot(N_+ + N_-)\cdot\tau, \tag{24}$$

$$\frac{N_+}{N_+ + N_-}\cdot\frac{\overline{\Delta V}}{q\overline{V_c}\tau} > 1, \tag{25}$$

Since the expression $\frac{N_+}{N_+ + N_-}$ is a positive probability of success in relative units, then replace it with p:

$$p\cdot\frac{\overline{\Delta V}}{q\overline{V_c}\tau} > 1, \tag{26}$$

After all transformations we get the following condition:

$$p\overline{\Delta V} > q\overline{V_c}\tau, \tag{27}$$

Thus, inequality (27) proves that a commercial bank will benefit from a start-up loan only when the probability of a successful completion of the start-up by the average amount of return to the financial institution after the successful completion of the start-up is greater than the profit received from the provision of a regular loan transaction, taking into account the time factor.

Let's introduce a decision on whether to provide a small business loan or not to start-up financing as a static theoretical game model depicted on Figure 2, and we will prove that the choice by a financial institution to provide start-up financing is Nash equilibrium [23].

	provide financing	refuse financing
successful	$p\overline{\Delta V}$ Nash equilibrium V_1	$-\left(p\overline{\Delta V}\right)'$ V_1
not successful	$-\left(q\overline{V_c}\tau + \overline{V_c}'\right)$ γ	$\overline{V_c}$ 0

Figure 2. Theoretically-gaming model for optimizing the interaction of banking financial structures and small businesses

Nash equilibrium was chosen as the game theory tool, since it is a method of self-regulation of systems of any degree of complexity, in contrast to the optimum for Paretto, which is used as a tracking apparatus [20, 23]. Consequently, on Figure 2 vertically we see two possible solutions of a financial institution (commercial bank) – to provide financing, or to refuse it. For its part, the firm-innovator also has two possible final variants of its start-up project: the project can be successful and the firm-innovator will receive profit V_1 or V_2, whether the project can be unsuccessful and the firm-innovator will go bankrupt and will be eliminated. In this case, we consider small firm-innovators who either do not have the equity to implement their idea, or it is so small that it will not be able to strengthen the financial position of the company in case of the failure of the project. That is, it is about the firms-innovators share of the borrowed capital in which is from 90% and above. In the event that the firm had a successful start-up project, as already mentioned above, it receives profits V_1 or V_2. It should be noted that the following condition is fulfilled:

$$V_2 \le V_1, \tag{28}$$

This discrepancy arises because if the first financial institution to which the innovative firm applied, decided not to finance the submitted project and the firm-innovator was forced to seek funding in another institution, then it spends extra time and unnecessary efforts. Also, it should be noted that the profit received by the innovative company V_1 is not the total profit from successful start-up project, because the condition is fulfilled:

$$V_1 = V_{tot.} - \overline{\Delta V}\,, \tag{29}$$

where $V_{tot.}$ is a total profit, and V_1 the difference between it and the share of profit received by the financial structure that provided the funds for the project.

It should be noted that when financing start-up projects funds are provided by various commercial financial institutions, as a rule, not a percentage of the body of the loan, and as a partner of the project on the right to participate in profits [5]. This is due to the fact that the risk of failure of start-up projects is more than 95%, but successful projects are so financially successful that offset not only failed projects but also bring significant financial gains that are not commensurate with the initial financial cost [5, 16].

In case of failure of the start-up project, when receiving funding, the firm-innovator bankrupt and receives losses in the amount of γ, the share returned to the bank in the event of the liquidation of the enterprise. When refusing funding, the firm-innovator does not lose anything, because the project was not launched.

If we consider the possibility of a decision by the commercial financial structure, in the case of financing provided the start-up project succeeds, the commercial bank receives the average profit $\overline{\Delta V}$ from the start-up for the entire period of existence of start-up with the probability *p*. In this case, as an average profit is considered profit, which is calculated as the arithmetic mean of the profits received from successful projects financed by this bank. If a commercial bank refuses to finance a start-up project, in case of its success, the financial institution has a loss of profit in the amount calculated by the formula:

$$-(p\overline{\Delta V})' = -p\overline{\Delta V}\,, \tag{30}$$

If, however, a financial institution has preferred ordinary lending for a given percentage, then the loss of profit should be adjusted for the profit derived from the loan, namely:

$$-(p\overline{\Delta V})' = -p\overline{\Delta V} + q\overline{V_c}\tau\,, \tag{31}$$

If the financing of a start-up project is provided by a financial institution in the event of its failure, the commercial bank has losses in the form of interest that would have been received in the provision of ordinary loans and adjusted for the share γ returned to the enterprise in the event of the bankruptcy of the firm-innovator:

$$\overline{V_c}' = \overline{V_c} - \gamma\,, \tag{32}$$

In case of failure of the commercial institution to finance the project failure, the financial institution receives as a result the distorted losses in the form of the loan body $\overline{V_c}$.

It should be noted that it $\overline{V_c}$ is calculated as the arithmetic mean of all commercially-funded (and successful and unsuccessful) start-ups of small businesses financed by the given commercial bank for a definite period equal to the period used to calculate the average profit. Given the Nash equilibrium, as shown in Figure 1, this equilibrium is present only in one case – when the financial institution agrees to finance the project with the probability of success *p*. According to the theory of games, this is precisely the situation that is optimal for self-regulation of the interaction of financial structures and start-up in small business

Taking into account expressions (15) – (17), the dependence (14) acquires the following finite form:

$$\mathrm{FS_{IAE}} = f(\alpha_1;\alpha_2;\alpha_3;\beta_1;\beta_2;\beta_3;\beta_4;\gamma_1;\gamma_2;\gamma_3) \rightarrow \text{argmax} \tag{33}$$

Since the factors in equation (33) are equivalent to the possibility of their inclusion, and in this model does not take into account the multiplicative and synergistic effects, as well as ignores the time factor in the form of a discount multiplier, then the model will be considered static rather than dynamic.

For simplicity of practical application, taking into account the foregoing, as an example, consider the linear dependence option for the developed multifactorial model, where the above factors are represented as variables (*x*), and the model of financial support of innovative activity of small enterprises in the form of an integral parameter (F_S):

$$F_s = \mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 + \mathrm{x}_4 + \mathrm{x}_5 + \mathrm{x}_6 + \mathrm{x}_7 + \mathrm{x}_8 + \mathrm{x}_9 + \mathrm{x}_{10} \quad (34)$$

The proposed linear model has a limited range of applications, but is simpler in calculations.

Thus, the developed model of financial support for innovative activity of small enterprises is formed taking into account ten factors that are included in the ratio (34). Since the developed model is a multi-factor group dependence, we interpret formula (34) taking into account the hierarchy and the three groups of factors presented in Figure 2:

$$F_s = x_1^1 + \mathrm{x}_2^1 + \mathrm{x}_3^1 + \mathrm{x}_1^2 + \mathrm{x}_2^2 + \mathrm{x}_3^2 + \mathrm{x}_4^2 + \mathrm{x}_1^3 + \mathrm{x}_2^3 + \mathrm{x}_3^3, \quad (35)$$

In general, the developed model is mobile, and the information in it is consolidated, from the lower levels, which allows it to quickly adapt it in the changing conditions of the modern market and information environment by adding both vertical and horizontal new significant factors. For simplicity of practical application, taking into account that the developed model does not take into account multiplicative and synergistic effects, and also ignores the time factor, we will consider the model as static rather than dynamic and consider the variant of linear dependence for the developed multi-factor model. As the factors in equation (35) are varied in their prevalence, availability and efficiency, we introduce weight coefficient k and represent formula (36) in the form of the following linear dependence:

$$F_s = x_1^1 \cdot \mathrm{k}_1^1 + \mathrm{x}_2^1 \cdot \mathrm{k}_2^1 + \mathrm{x}_3^1 \cdot \mathrm{k}_3^1 + \mathrm{x}_1^2 \cdot \mathrm{k}_1^2 + \mathrm{x}_2^2 \cdot \mathrm{k}_2^2 + \mathrm{x}_3^2 \cdot \mathrm{k}_3^2 + \mathrm{x}_4^2 \cdot \mathrm{k}_4^2 + \mathrm{x}_1^3 \cdot \mathrm{k}_1^3 + \mathrm{x}_2^3 \cdot \mathrm{k}_2^3 + \mathrm{x}_3^3 \cdot \mathrm{k}_3^3, \quad (36)$$

Using the above methodology for selecting experts (formulas (3) – (6)) and the method of active examination, an expert survey for the determination of *q (π(s))* on the quantitative significance of the weight coefficient $\mathrm{k}_\mathrm{j}^\mathrm{i}$ for both the Ukrainian conditions (21 experts) and for China(23 experts) was conducted, the results of which are presented in (Table 1) and in (Table 2).

Table 1. Expert evaluation of factors significance for Ukraine

Experts (21 expert)	factors										General significance, %
	x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_4^2	x_1^3	x_2^3	x_3^3	
agreed indicator value $\mathrm{k}_\mathrm{j}^\mathrm{i}$	58,000	5,000	8,000	1,000	3,000	1,500	9,000	4,500	3,000	7,000	*100,0*

Table 2. Expert evaluation of factors significance for China

Experts (23experts)	factors										General significance, %
	x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_4^2	x_1^3	x_2^3	x_3^3	
agreed indicator value k_j^i	5,000	9,000	5,000	23,000	19,000	18,000	4,000	6,000	5,000	6,000	*100,0*

To verify the correctness of the expert data, the calculation of the coefficient of concordance by formulas (9) and (10) was performed:

$$w^{UA} = \frac{12 \times 29422,5}{21^2 \times (10^3 - 10)} = \frac{353070}{436590} = 0,8 \; w^{CH} = \frac{12 \times 37096,5}{23^2 \times (10^3 - 10)} = \frac{445158}{523710} = 0,85$$

The results obtained suggest that the consistency of expert opinions for both Ukraine and China is satisfactory, since the obtained result is more than 0.35 [21], and therefore the results of expert research are correct. Based on the calculations

made by the formulas, the error does not go beyond the recommended, and therefore the results of the study can be considered reliable and correct (Table 3) and (Table 4).

Table 3. Results of error calculation for k_j^i (Ukraine)

Factor	x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_4^2	x_1^3	x_2^3	x_3^3
$\overline{X}$	58,000	5,000	8,000	1,000	3,000	1,500	9,000	4,500	3,000	7,000
σ^2	1,333	0,476	0,571	0,006	0,143	0,052	0,857	0,232	0,090	0,420
σ %	1,99	13,80	9,45	7,56	12,60	15,26	10,29	10,71	9,97	9,26

Table 4. Results of error calculation for k_j^i (China)

Factor	x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_4^2	x_1^3	x_2^3	x_3^3
$\overline{X}$	5,000	9,000	5,000	23,000	19,000	18,000	4,000	6,000	5,000	6,000
σ^2	0,076	0,444	0,104	0,648	0,764	0,353	0,157	0,181	0,172	0,397
σ %	5,5	7,4	6,45	3,5	4,6	3,3	9,9	7,1	8,3	10,5

Substituting the equations obtained for all factors, both for the first and for the second order (for franchising), and for experts calculated by the value *k* in formula (36), we obtain the current model for financial support of innovative activity of small business (37) for Ukraine today:

$$F_s = x_1^1 \cdot 0,58 + x_2^1 \cdot 0,05 + x_3^1 \cdot 0,08 + x_1^2 \cdot 0,01 + x_2^2 \cdot 0,03 + x_3^2 \cdot 0,015 + \\ + x_4^2 \cdot 0,09 + x_1^3 \cdot 0,045 + x_2^3 \cdot 0,03 + x_3^3 \cdot 0,07(x_3^{3.1} \cdot 0,6 + x_3^{3.2} \cdot 0,4) \quad (37)$$

It should be noted that the second-order values and the franchise factor (formula (21)) calculated by the experts have the following relevant averaging values: 0.6 and 0.4 for Ukraine; 0.2 and 0.8 for China. For the realities of China, this model has the following form:

$$F_s^{CH} = x_1^1 \cdot 0,05 + x_2^1 \cdot 0,09 + x_3^1 \cdot 0,05 + x_1^2 \cdot 0,23 + x_2^2 \cdot 0,19 + x_3^2 \cdot 0,18 + \\ + x_4^2 \cdot 0,04 + x_1^3 \cdot 0,06 + x_2^3 \cdot 0,05 + x_3^3 \cdot 0,06(x_3^{3.1} \cdot 0,8 + x_3^{3.2} \cdot 0,2) \quad (38)$$

Taking into account the experience of China and the predicted optimal value of all factors, we will translate in formula (34) an effective for Ukraine model of optimization of financial support of innovative business of small business:

$$F_s^{ef} = x_1^1 \cdot 0,05 + x_2^1 \cdot 0,09 + x_3^1 \cdot 0,06 + x_1^2 \cdot 0,2 + x_2^2 \cdot 0,16 + x_3^2 \cdot 0,18 + \\ + x_4^2 \cdot 0,05 + x_1^3 \cdot 0,07 + x_2^3 \cdot 0,06 + x_3^3 \cdot 0,08 \cdot (x_3^{3.1} \cdot 0,7 + x_3^{3.2} \cdot 0,3) \quad (39)$$

In general, the model represented in formula (36) can be adapted to any country in conducting relevant expert studies. For Ukraine, the practical model is given in the formula (39), the use of which will yield a significant economic effect both at the micro and macro levels.

5. CONCLUSION

Having analyzed the existing ways of providing financial support for innovative activities of small enterprises, which in our opinion are the most widespread today, a number of the most important factors, which at the same time are sources of financial support of innovative activity of small business, and graphically reflected as a model of financial provision of innovative activity of small Business (FS_{IAE}) on Figure 1. A mathematically-developed model (F_S) is described by a number of formulas presented in the article. To simplify the practical application, given that the

developed model does not take into account the multiplicative and synergistic effects, and also ignores the time factor, the model is considered as static rather than dynamic and considers the linear dependency option for the developed multifactorial model. Given that the factors that are included in the functional dependence of F_S are varied in their weight, a weighting factor *k* was introduced, to determine the magnitude of which the active examination method was used, and to verify the accuracy of the expert data, the calculation of the coefficient of concordance and the calculation of the statistical mistakes Taking into account the value of experts calculated by experts, the actual model for financial support of innovative activity of small enterprises for Ukraine has been received for today. Given the experience of China and the effective value of the indicators, predicted by experts, an model for optimizing the financial support of innovative activity of small enterprises was developed, the use of which will produce a significant economic effect both at micro and macro levels. Also, for the factor $\beta 4$ – bank loans, the theoretical-game model of interaction between financial structures and subjects of innovative activity was developed, which proves that, under the conditions defined in the model, financial structures have advantageous financing of innovative start-ups of small businesses, as this is the balance of Nash.

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